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SECTION OF

Mathematical Sciences

(Including Statistics)

President

A.K. Agarwal

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Presidential Address

President

A.K. Agarwal

Ramanujan's Last Discovery – The Mock Theta Functions

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Distinguished invited speakers, fellow mathematicians, ladies and gentlemen!

At the outset I would like to thank the members of ISCA for electing me as President of the Section of Mathematical Sciences (including statistics) of the Indian Science Congress Association for the year 2009 - 10. I am indeed deeply humbled by my election and assure you all that it would be my constant endeavour during my tenure and thereafter to serve ISCA to the best of my abilities. We are very happy to be here in the God's own country.

As a tribute to the India's most legendary mathematician Srinivasa Ramanujan, I will be talking about his last discovery- the mock theta functions.

Srinivasa Ramanujan is regarded as the most influential Indian Mathematician of the twentieth century. The remarkable thing about him was: without any formal training and in the midst of extreme poverty he was able to do original mathematics research of the highest quality. He did not give up even in the face of his death. Early in 1920, three months before his death, he wrote his last letter to Hardy. In the course of it he said: "I discovered very interesting functions recently which I call 'mock θ- functions', they enter into mathematics as beautifully as the ordinary θ- functions". He enclosed with his letter three lists of 17 functions. First containing 4 functions of order 3, second containing 10 functions of order 5 and the third containing 3 functions of order 7. Watson [20] found three more functions of order 3 and two more of order 5 appear in the 'lost' notebook [18]. Mock theta functions of order 6, 8 and 10 have also been studied in [12],[17] and [14], respectively.

Definition 1.1.

[19]. A mock - theta function is a function defined by a q - series convergent when $|q| < 1$, for which we can calculate asymptotic formulae when q tends to a 'rational point' $e^{2\pi r\, t/s}$ of the unit circle of the same degree of precision as those furnished for the ordinary θ- function by theory of linear transformation.

Definition 1.2.

[8]. The order of a mock-theta function is $(2r+1)$ if it is expressible in terms of $a\ r+1\phi_r$ series, on a single base q^k, $k \leq r+1$. There may be in addition of the mock-theta functions an additive term with $r+1\phi_r$ consisting of θ-products, which do not effect the order. It is understood that $r+1\phi_r$ is expressed in terms of the lowest possible order θ - function.

Remark

With this criterion, the labelling of all the third, fifth and seventh order mock - theta functions is complete because the seven mock - theta functions of order 3 are defined by certain $2\phi_1$ series with the possible addition of certain 'trivial' functions, such as theta function products. The series are defined on a single base either q or q^2. Similarly, the ten mock-theta functions of order five have been defined as the limits of certain $3\phi_2$ series on a single base q or q^2, with the possible additional term which is a 'trivial' function, if at all. These $3\phi_2$ functions can not be reduced to a $2\phi_1$ series by any transformation. Also, all the three mock-theta functions of order seven, are expressible as limits of 'irreducible' $4\phi_3$ series on a single base q.

Difficulty arises in the 'order' labelling of mock - theta functions of recent origin. Still today the question of giving a formal definition of the order of mock - theta function is not finally settled.

The rising q-factorial is defined by

$$(a\,;q)_n = \prod_{i=0}^{\infty} \frac{(1-aq^{i})}{(1-aq^{n+i})}\ \text{, for any constant } a.$$

In n is a positive integer, then obviously

$$(a\,;q)_n = (1-a)(1-aq)\ldots\ldots\ldots\ldots(1-aq^{n-1}),$$

and

$$(a\,;q)_\infty = (1-a)(1-aq)\ldots\ldots\ldots\ldots(1-aq^{2})\ldots\ldots\ldots\ldots$$

A series involving factors of the forms $(a;q)_n$, is called a q - series or basic series or Eulerian series.

The generalized basic hypergeometric series is defined as

$$ {}_r\phi_s\begin{pmatrix} a_1 & a_2 & \dots & a_r ; z \\ b_1 & b_2 & \dots & b_r \end{pmatrix} = \sum_{n=0}^{\infty} \frac{(a_1;q)_n (a_2;q)_n \dots\dots (a_r;q)_n [(-1)^n q^{(1/2)n(n-1)}]^{1+s-r} z^n}{(b_1;q)_n (b_2;q)_n \dots\dots (b_s;q)_n (q;q)_n}, $$

where $q \neq 0, r > s+1$.

Mock theta functions of order 3 are:

$$ \mathrm{f}(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q;q^2)_n}, \qquad (1.1) $$

$$ \phi(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q^2;q^2)_n}, \qquad (1.2) $$

$$ \psi(q) = \sum_{n=1}^{\infty} \frac{q^{n^2}}{(q;q^2)_n} \qquad (1.3) $$

and

$$ \chi(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(1-q+q^2)(1-q^2+q^4)\dots\dots\dots(1-q^n+q^{2n})}. \qquad (1.4) $$

Mock theta functions of order 5 are:

Group – A

$$ \mathrm{f}_0(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q;q)_n}, \qquad (1.5) $$

$$ \phi_0(q) = \sum_{n=0}^{\infty} q^{n^2} (-q;q^2)_n, \qquad (1.6) $$

$$ \psi_0(q) = \sum_{n=0}^{\infty} q^{(n+1)(n+2)/2} (-q;q)_n, \qquad (1.7) $$

$$ F_0(q) = \sum_{n=0}^{\infty} \frac{q^{2n^2}}{(q;q^2)_n}, \qquad (1.8) $$

and

$$ \chi_0(q) = \sum_{n=0}^{\infty} \frac{q^n}{(q^{n+1};q)_n}. \qquad (1.9) $$

Group – B

$$f_1(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(-q;q)_n}, \qquad (1.10)$$

$$\phi_1(q) = \sum_{n=0}^{\infty} q^{(n+1)^2}(-q;q^2)_n \;, \qquad (1.11)$$

$$\psi_1(q) = \sum_{n=0}^{\infty} q^{n(n+1)}(-q;q)_n \;, \qquad (1.12)$$

$$F_1(q) = \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(q;q^2)_{n+1}}, \qquad (1.13)$$

and

$$\chi_1(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^{n+1};q)_{n+1}}. \qquad (1.14)$$

And the mock theta functions of order 7 are:

$$\Im_0(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^{n+1};q)_n}, \qquad (1.15)$$

$$\Im_1(q) = \sum_{n=1}^{\infty} \frac{q^{n^2}}{(q^n;q)_n}, \qquad (1.16)$$

and

$$\Im_2(q) = \sum_{n=1}^{\infty} \frac{q^{n^2+n}}{(q^{n+1};q)_{n+1}}. \qquad (1.17)$$

Watson [20] found three more functions of order 3,

$$\omega(q) = \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(q;q^2)_{n+1}}, \qquad (1.18)$$

$$\nu(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(-q;q^2)_{n+1}}, \qquad (1.19)$$

and $$\rho(q) = \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(1+q+q^2)(1+q^3+q^6)..........(1+q^{2n+1}+q^{4n+2})} \qquad (1.20)$$

Mock theta functions have been studied in different directions. For example, Ramanujan gave, without proof, the following identities satisfied by the functions in Group-A of mock theta functions of order 5:

$$\phi_0(-q)+\chi_0(q)=2F_0(q), \tag{1.21}$$

$$\begin{aligned} \mathrm{f}_0(q)+2F_0(-q^2)-2 &= \phi_0(-q^2)+\chi_0(-q) \\ &= 2\phi_0(-q^2)-\mathrm{f}_0(q) \\ &= \theta_4(0,q)[(q;q^5)_\infty(q^4;q^5)_\infty]^{-1}, \end{aligned} \tag{1.22}$$

$$\psi_0(q)-F_0(q^2)+1=\frac{q(q^4;q^4)_\infty}{(q^2;q^2)_\infty}[(q^8;q^{20})_\infty(q^{12};q^{20})_\infty]^{-1}. \tag{1.23}$$

He further proved that the Group-B of fifth order mock theta functions also satisfies similar relations given by

$$\psi_1(q)-q^{-1}\phi_1(-q)=2F_1(q), \tag{1.24}$$

$$\begin{aligned} \mathrm{f}_1(-q)+2qF_1(q^2) &= q^{-1}\phi_1(-q^2)+\psi_1(-q) \\ &= 2q^{-1}\phi_1(-q^2)+\mathrm{f}_1(q) \\ &= \theta_4(0,q)[(q^2;q^5)_\infty(q^3;q^5)_\infty]^{-1} \end{aligned} \tag{1.25}$$

$$\chi_1(q)-qF_1(q^2)=\frac{(q^4;q^4)_\infty}{(q^2;q^4)_\infty}[(q^4;q^{20})_\infty(q^{16};q^{20})_\infty]^{-1}, \tag{1.26}$$

where $\theta_4(z,q)$ is the Jacobi's theta function, defined by

$$\theta_4(z,q)=\prod_{n=1}^{\infty}(1-q^{2n})(1-q^{2n-1}\cos 2z+q^{4n-2}).$$

Watson [21] proved all the identities given above by the methods of rearrangement of series. He also made use of the basic hypergeometric series for finding new representation of third order mock theta functions given by (1.1) – (1.4) and (1.18) -(1.20) in the following forms:

$$\mathrm{f}(q)\prod_{n=1}^{\infty}(1-q^n)=1+4\sum_{n=1}^{\infty}\frac{(-1)^n q^{n(3n+1)/2}}{1+q^n}, \tag{1.27}$$

$$\phi(q)\prod_{n=1}^{\infty}(1-q^n)=1+2\sum_{n=1}^{\infty}\frac{(-1)^n(1+q^n)q^{n(3n+1)/2}}{1+q^{2n}}, \tag{1.28}$$

$$\chi(q)\prod_{n=1}^{\infty}(1-q^n)=1+\sum_{n=1}^{\infty}\frac{(-1)^n(1+q^n)q^{n(3n+1)/2}}{1-q^n+q^{2n}}, \qquad (1.29)$$

$$\psi(q)\prod_{n=1}^{\infty}(1-q^n)=\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{6n(n+1)+1}}{1-q^{4n+1}}, \qquad (1.30)$$

$$\omega(q)\prod_{n=1}^{\infty}(1-q^{2n})=\sum_{n=0}^{\infty}\frac{(-1)^n q^{3n(n+1)}(1+q^{2n+1})}{1-q^{2n+1}}, \qquad (1.31)$$

$$\nu(q)\prod_{n=1}^{\infty}(1-q^n)=\sum_{n=0}^{\infty}\frac{(-1)^n q^{3n(n+1)/2}(1-q^{2n+1})}{1+q^{2n+1}}, \qquad (1.32)$$

and

$$\rho(q)\prod_{n=1}^{\infty}(1-q^{2n})=\sum_{n=1}^{\infty}\frac{(-1)^n q^{3n(n+1)/2}(1-q^{4n+1})}{(1+q^{2n+1}+q^{4n+2})}, \qquad (1.33)$$

Andrews ([10, 11]) gave a set of very elegant basic hypergeometric transformations which yielded as special cases the mock- theta functions relations.

A partition of a positive integer η is a non- increasing sequence of positive integers whose sum is η 0 also has a partition called "empty partition". The rank of a partition is defined to be the largest part minus the number of its parts. Partition theoretic interpretations of some of the mock theta functions are found in the literature. For example, N.J. Fine [15] interpreted $\psi(q)$ - a mock theta function of order 3 as generating function for partitions into odd parts without gaps. And George Andrews [9] interpreted $F_0(q)$ as generating function for partitions into odd parts without gaps and each part appears at least two times.

Below are given computer produced tables of $\psi(q)$, $F_o(q)$, $\phi_0(q)$, $\phi_1(q)$ and $F_1(q)$:

$\sum_{n=1}^{\infty} O(n)q^n$, where $O(n)$ is the number of partitions on n into odd parts

$$1.\ \sum_{n=1}^{\infty} O(n)q^n = q+q^2+2q^3+2q^4+3q^5+4q^6+5q^7+6q^8+8q^9 +10q^{10}+12q^{11}+15q^{12}+..$$

2. $\psi(q) = q+q^2+q^3+2q^4+2q^5+2q^6+3q^7+3q^8+4q^9$
$+5q^{10}+5q^{11}+6q^{12}+7q^{13}+8q^{14}+9q^{15}+11q^{16}+12q^{17}+13q^{18}+\ldots.$

3. $F_0(q) = 1+q^2+q^3+q^4+q^5+q^6+q^7+2q^8+2q^9$
$+2q^{10}+3q^{11}+3q^{12}+3q^{13}+4q^{14}+4q^{15}+4q^{16}+5q^{17}+6q^{18}+\ldots.$

4. $\phi_0(q) = 1+q+q^2+q^4+q^5+q^7+q^8+q^9$
$+q^{10}+q^{12}+q^{13}+q^{14}+q^{15}+q^{16}+2q^{17}+q^{18}+\ldots.$

5. $\phi_1(q) = q+q^4+q^5+q^9+q^{10}+q^{12}+q^{13}+q^{16}+q^{17}+q^{19}+\ldots..$

6. $F_1(q) = 1+q+q^2+q^3+2q^4+2q^5+2q^6+3q^7+3q^8+3q^9$
$+4q^{11}+5q^{12}+6q^{13}+6q^{14}+\ldots.$

In the sequel we shall discuss combinatorial interpretations of some of the mock theta functions found recently.

2. Partitions with "*n*+*t* copies of *n* " and mock theta functions

Definition 2.1 (Agarwal and Andrews, 1987 [3]).

A partition with "*n*+*t* copies of *n*", t ≥ 0 is a partition in which a part of size, *n* ≥ 0, can come in a n+t different colours denoted by subscripts: a $n_1, n_2, \ldots, n_{n+t}$

Thus, for example, the partitions of 2 with "*n*+1 copies of *n*" are

2_1,	2_1+0_1,	1_1+1_1,	$1_1+1_1+0_1$
2_2,	2_2+0_1,	1_2+1_1,	$1_2+1_1+0_1$
2_3,	2_3+0_1,	1_2+1_2,	$1_2+1_2+0_1$.

Note that zeros are permitted if and only if t is greater than or equal to one. Also, in no partition are zeros permitted to repeat.

Partitions with "*n* copies of *n*", are also called *n*-colour partitions and arise in Baxter's solution of the hard hexagon model.

Definition 2.2 (Agarwal and Andrews, 1987 [3]).

The weighted difference of two parts m_i, n_j, $m \geq n$ is defined by $m - n - i - j$ and denoted by $((m_i - n_j))$.

Agarwal, 2004 [1] interpreted mock theta function $\psi(q)$, $F_0(q)$, $\phi_0(q)$ and $\phi_1(q)$ combinatorially, using n- colour partitions. Following are his results:

Theorem 2.1.

For $\nu \geq 1$, let $A_1(\nu)$ denote the number of n-colour partitions of ν such that even parts appear with even subscripts and odd with odd, for some k, k_k is a part, and the weighted difference of any two consecutive parts is 0. Then

$$\sum_{\nu=1}^{\infty} A_1(\nu) q^{\nu} = \psi(q). \qquad (2.1)$$

Example. $A_1(8) = 3$. The relevant n- colour partitions are: 8_8, 7_5+1_1, 6_2+2_2.

Theorem 2.2.

For $\nu \geq 0$, let $A_2(\nu)$ denote the number of n-colour partitions of ν such that even parts appear with even subscripts and odd with odd greater than 1, for some k, k_k is a part, and the weighted difference of any two consecutive parts is 0. Then

$$\sum_{\nu=0}^{\infty} A_2(\nu) q^{\nu} = F_0(q). \qquad (2.2)$$

Theorem 2.3.

For $\nu \geq 0$, let $A_3(\nu)$ denote the number of n-colour partitions of ν such that only the first copy of the odd parts and the second copy of the even parts are used, that is, the parts are of the type $(2k-1)_1$ or $(2k)_2$, the minimum part is 1_1 or 2_2, and the weighted difference of any two consecutive parts is 0. Then

$$\sum_{\nu=0}^{\infty} A_3(\nu) q^{\nu} = \phi_0(q). \qquad (2.3)$$

Theorem 2.4.

For $\nu \geq 1$, let $A_4(\nu)$ denote the number of n-colour partitions of ν such that only the first copy of the odd parts and the second copy of the even parts are used, the minimum part is 1_1, and the weighted difference of any two consecutive parts is 0. Then

$$\sum_{\nu=1}^{\infty} A_4(\nu)q^{\nu} = \phi_1(q). \qquad (2.4)$$

Remark.

We remark that there are 160 *n*- colour partititons of 8 but only one partition, viz., 6_2+2_2 is relevant for Theorem 2.3 and none is relevant for Theorem 2.4. Out of 859 *n*-colour partitions of 11, none is relevant for Theorems 2.3-2.4. Among 18334 *n* -colour partitions of 17 only two viz., $9_1+6_2+2_2$ and $8_2+5_1+3_1+1_1$ satisfies the conditions of Theorem 2.3, where as the lone partition $8_2+5_1+3_1+1_1$ satisfies the condition of Theorem 2.4.

Very recently Agarwal and Rana, 2008 [6] interpreted another 5th order mock theta function $F_1(q)$ combinatorially by using partitions with "*n*+2 copies of *n*" in the following form:

Theorem 2.5.

For $\nu \geq 0$, let $A_5(\nu)$ denote the number of partitions of ν with "*n*+2 copies of *n*" in which even parts appear with even subscripts and odd with odd greater than 1. For some *i*, i_{i+2} is a part and the weighted difference of any two
j consecutive parts is zero. Then

$$\sum_{\nu=0}^{\infty} A_5(\nu)q^{\nu} = F_1(q). \qquad (2.5)$$

Example. $A_5(7) = 3$, since the relevant (*n*+2) – colour partitions are 7_9, 7_5+0_2, 6_2+1_3.

3. Lattice paths and mock theta functions

Agarwal and Bressoud [4] studied a new class of weighted lattice paths and used them to interpret several *q*- series combinatorially. Recently in [2, 6] these lattice paths have been used to provide new combinatorially interpretations of five mock theta functions discussed in the previous section. First we recall the following description of lattice paths from [4].

All paths will be of finite length lying in the first quadrant. They will begin on the y-axis and terminate on the x-axis. Only three moves are allowed at each step:

northeast : from (i, j) to $(i+1, j+1)$,

southeast : from $(i,\ \ )$ to $(i+1, j-1)$, only allowed if $j > 0$,

horizontal: from $(i, 0)$ to $(i+1, 0)$, only allowed along x-axis.

All our lattice paths are either empty or terminate with a southeast step: from $(i,1)$ to $(i+1, 0)$.

The following terminology will be used in describing lattice paths:

Peak

Either a vertex on the y-axis which is followed by a southeast step or a vertex preceded by a northeast step and followed by a southeast step.

Valley

A vertex preceded by a southeast step and followed by a northeast step. Note that a southeast step followed by a horizontal step followed by a northeast step does not constitute a valley.

Mountain

A section of the path which start on either the x-axis or y-axis, which ends on the x-axis, and which does not touch the x-axis anywhere in between the end points. Every mountain has at least one peak and may have more than one.

Plain

A section of the path consisting of only horizontal steps which starts either on the y-axis or at a vertex preceded by a southeast step and ends at a vertex Agarwal in [2] translated Theorem 2.1 – 2.4 for lattice paths as follows: followed by a northeast step.

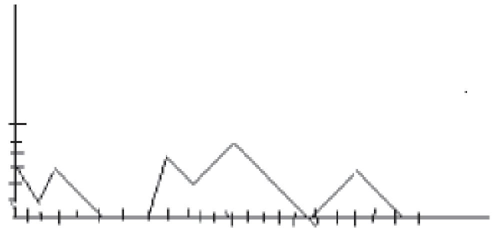
Height of a vertex is its y-coordinate. The **Weight of a vertex** is its x-coordinate. The **Weight of a path** is the sum of the weights of its peaks.

Theorem 3.1.

For $\nu \geq 1$, let $B_1(\nu)$ denote the number of lattice paths of weight ν which start from (0, 0), have no valley above height 0 and no plain. Then

$$\sum_{\nu=1}^{\infty} B_1(\nu) q^{\nu} = \psi(q). \qquad (3.1)$$

Example. The following path has 5 peaks, 3 valleys, 3 mountains and 1 plain.



Theorem 3.2.

For $\nu \geq 0$, let $B_2(\nu)$ denote the number of lattice paths of weight ν which start from (0,0), have no valley above height 0, no plain and the height of each peak is ≥ 2. Then

$$\sum_{\nu=0}^{\infty} B_2(\nu)q^{\nu} = F_0(q). \tag{3.2}$$

Theorem 3.3.

For $\nu \geq 0$, let $B_3(\nu)$ denote the number of lattice paths of weight ν which start from (0,0), have no valley above height 0, no plain, the height of each peak of odd weight is 1 while that of even weight is 2. Then

$$\sum_{\nu=0}^{\infty} B_3(\nu)q^{\nu} = \phi_0(q). \tag{3.3}$$

Theorem 3.4.

For $\nu \geq 1$, let $B_4(\nu)$ denote the number of lattice paths of weight ν which start from (0,0), have no valley above height 0, no plain, the height of each peak of odd weight is 1 while that of even weight is 2 and the weight of the first peak is 1. Then

$$\sum_{\nu=1}^{\infty} B_4(\nu)q^{\nu} = \phi_1(q). \tag{3.4}$$

Very recently, Agarwal and Rana [6] succeeded in translating Theorem 2.5 also for the lattice paths in the following form:

Theorem 3.5.

For $\nu \geq 0$, let $B_5(\nu)$ denote the number of lattice paths of weight ν which start at (0,2), have no valley above height 0, no plain, and for which the height of each peak is ≥ 2. Then

$$\sum_{\nu=0}^{\infty} B_5(\nu)q^{\nu} = F_1(q). \tag{3.5}$$

4. Frobenius partitions and mock theta functions

A two – rowed array of non- negative integers

$$\begin{pmatrix} a_1 & a_2 & \dots & a_r \\ b_1 & b_2 & \dots & b_r \end{pmatrix}$$

$a_1 \geq a_2 \geq \dots \geq a_r \geq 0$; $b_1 \geq b_2 \geq \dots \geq b_r \geq 0$, is known as a generalized Frobenius partition or more simply an F- partition of n if

$$n = r + \sum_{i=1}^{r} a_i + \sum_{i=1}^{r} b_i .$$

These partitions are attributed to Frobenius [16] because he was the first to study them in his work on group representation theory under the additional assumption,

$$a_1 > a_2 > \ldots > a_r > 0; \qquad b_1 > b_2 > \ldots > b_r > 0.$$

Recently Agarwal and Narang [5] using Frobenius partitions interpretated the mock theta function $\psi(q)$, $F_0(q)$, $\phi_0(q)$ and $\phi_1(q)$ combinatorially as follows:

Theorem 4.1.

For $\nu \geq 1$, let $C_1(\nu)$ denote the number of Frobenius partitions of ν such that

(4.1.a) $a_i \geq b_i$, $1 \leq i \leq r$,

(4.1.b) $b_i = a_{i+1}$, + 1, $1 \leq i \leq r-1$ and

(4.1.c) $b_r = 0$.

Then $\sum_{\nu=1}^{\infty} C_1(\nu) q^{\nu} = \psi(q)$. (4.1)

Theorem 4.2.

For $\nu \geq 0$, let $C_2(\nu)$ denote the number of Frobenius partitions of ν such that

(4.2.a) $a_i \geq b_i$, $1 \leq i \leq r$,

(4.2.b) $b_i = a_{i+1} + 1$, $1 \leq i \leq r-1$,

(4.2.c) $b_r = 0$ and

(4.2.d) $a_r \neq 0$.

Then $\sum_{\nu=0}^{\infty} C_2(\nu) q^{\nu} = F_0(q)$. (4.2)

Theorem 4.3.

For $\nu \geq 0$, let $C_3(\nu)$ denote the number of Frobenius partitions of ν such that

(4.3.a) $a_i = b_i$ or $a_i = b_i+1$, $1 \le i \le r$,
(4.3.b) $b_i = a_{i+1}+1$, $1 \le i \le r-1$ and
(4.3.c) $b_r=0$.

Then $$\sum_{\nu=0}^{\infty} C_3(\nu)q^{\nu} = \phi_0(q). \qquad (4.3)$$

Theorem 4.4.

For $\nu \geq 1$, let $C_4(\nu)$ denote the number of Frobenius partitions of ν such that

(4.4.a) $a_i = b_i$ or $a_i = b_i+1$, $1 \le i \le r$,
(4.4.b) $b_i = a_{i+1}+1$, $1 \le i \le r-1$ and
(4.4.c) $a_r = b_r=0$.

Then $$\sum_{\nu=0}^{\infty} C_4(\nu)q^{\nu} = \phi_1(q). \qquad (4.4)$$

Agarwal and Rana [7] have provided Frobenius partition theoretic interpretation of $F_1(q)$ as follows:

Theorem 4.5.

For $\nu \geq 0$, let $C_5(\nu)$ denote the number of Frobenius partitions of ν such that

(4.5.a) $a_r = 0$ or 2,
(4.5.b) $a_i \le b_i+1$, and
(4.5.c) $a_i = b_{i+1}+3$.

Then $$\sum_{\nu=0}^{\infty} C_5(\nu)q^{\nu} = F_1(q). \qquad (4.5)$$

5. New combinatorial identities

Theorems 2.1 – 2.5, 3.1 – 3.5 and 4.1 – 4.5 lead to the following 3 – way combinatorial identities:

$$A_i(\nu) = B_i(\nu) = C_i(\nu), \text{ for } 1 \le i \le 5. \quad (5.1)$$

6. Conclusion

Even after about 9 decades of its discovery the mock theta functions is still a very active area of research. Very recently, Bringmann and Ono [13] redefined mock theta functions as the holomorphic projections of weight ½ weak mass forms and used their ideas in solving the classical problem of obtaining formulas for $N_e(n)$ (resp. $N_0(n)$), the number of partitions of n with even (resp. odd) rank by showing the equivalence of this problem and the problem of deriving exact formulas for the coefficients α (n) of the series

$$f(q) = 1 + \sum_{n=1}^{\infty} \alpha(n) q^n ,$$

where $f(q)$ is the first mock theta function of order 3 defined by (1.1).

We have seen how Ramanujan's discovery of mock theta functions has inspired many mathematicians around the globe. We hope that it will continue to inspire generations to come.

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II

ABSTRACT OF

Platinum Jubilee Lecture

Complex Dynamics: An Overview

Anand P. Singh
Department of Mathematics
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Abstract

We give an introduction to complex dynamics and give a brief survey on the subject specifically dealing with the wandering domains, Fatou components, dynamics of composite entire functions, escaping sets of entire functions.

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III

ABSTRACTS OF

Young Scientist Award Programme

1. Discriminating Between the Bivariate Generalized Exponential and Bivariate Weibull Distribution

Arabin Kumar Dey[H] & Debasis Kunduh

H. Department of Mathematics and Statistics, Indian Institute of Technology Kanpur, Pin 208 016, India.

Key Words: **Bivariate Weibull, Bivariate Generalized Exponential, likelihood ratio test, EM algorithm, probability of correct selection**

Abstract

Bivariate Generalized Exponential and Bivariate Weibull distribution is often used to analyze two dimensional image processing data or a biological data or in general a failure time data. In this paper we discuss several issues of choosing the proper bi-variate distributions and critical situations arise on account of model mis-specification. We suggest maximum likelihood estimates for discrimination purpose. After highlighting the difficulties in estimation procedure we use EM algorithm to overcome them. Asymptotic distribution of the discrimination statistics has been derived to calculate the probability of correctly selecting the parent distribution. We perform some simulation study to verify our theoretical result. We illustrate our result by analyzing a real life data set.

2. *GαO-Kernel in the Digital Plane

R. Devi & M. Vigneshwaran

Department of Mathematics Kongunadu
Arts and Science College (Autonomous), Coimbatore-641 029, Tamilnadu, India.
E-mail: rdevicbe(@yahoo.com and vignesh.mat(@)gmail.com

Abstract

Digital topology was first studied in the late 1960's by the computer image analysis researcher Azriel Rosenfeld. The digital plane is a Mathematical model of the computer screen. In this paper we investigate explicit forms of $^*G\alpha$-kernel and $^*g\alpha$-closed sets in the digital plane. Also we prove that the digital plane is an ${}_{\alpha}T_{1/2}^{**}$ space.

3. On Super Edge Magic and Super (a, d)-Edge Antimagic Graphs

A. Saibulla[1] & P. Roushini Leely Pushpam[2]

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2. D.B. Jain College, Thorapakkam, Chennai-600 097
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Key Words: **Graph Labeling, Edge Sum, Vertex Sum, Graph join, Magic Labeling, Anti-magic Labeling, Fan Graph, Star Graph , Complete Bipartite Graph.**

Abstract

A *total labeling* of a graph *G* with *p* vertices and *q* edges is a one to one map taking the vertices and edges onto the set $\{1, 2, ..., p+q\}$. It is said to be *edge magic* if all the edges have same *edge sum* and *(a, d)-edge antimagic* if all the edges have distinct *edge sum* which are in arithmetic progression with

first term ‘a’ and common difference ‘d’. An *edge magic [(a, d)-edge antimagic]* is called a *super edge magic [super (a, d)-edge antimagic]* if the vertices are labeled to {1, 2, ..., *p*}. In this paper we prove that the *Fan graph* $F_{m,n,}$ *Bi-star* $B_{m,n}$ and the *extended Bi-star* $<K_{1,m} : n>$ are both *super edge magic* and *super (a, 2)-edge antimagic*. Also we have proved that a graph is super edge magic if and only if it is super (a, 2)-edge antimagic for some a.

4. Effect of Makoff Model on Combined Sampling Plans Under Inspection Error with Known Coefficient of Variation

Mujahida Sayyed
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Key Words: **OC, ASN, CV, AR(1), Inspection error, Sampling Plan**

Abstract

The Markoff model is examined to cast light on its physical interpretation and to facilitate its use. The purpose of this paper is, therefore, to determine and illustrate the effects of inspection error on the OC and ASN functions for independent and dependent mixed acceptance sampling plans. Where in variable sampling plans, random error terms are considered to be according to Markoff model for coefficient of variation (CV) and attribute sampling plans analysis with regard to the choice of a sampling plan taking inspection error into consideration. A comparison between the independent and dependent mixed plan have been made in respect of OC and ASN functions under inspection error. Result of the paper shows that the OC function decrease for independent and dependent mixed plans while for the independent mixed plan the ASN function decreases and for dependent mixed plan the ASN function are same for three different situations when compared with the error free case with inspection error for known CV.

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IV

ABSTRACTS OF

Symposia / Invited Lectures / Special Sessions

Symposium - 1

ALGEBRA AND CATEGORY THEORY

Organiser : Prof. A.R. Rajan

1. Generalized Regularities in Near-rings

T. Tamizh Chelvam

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Manonmaniam Sundarnar University
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Abstract

Regularity conditions are generalizations of inverse property with respect to multiplication in a ring. There are several characterizations through regularities for rings to be fields. Near-rings are generalized rings. For basic definitions on near-rings, one may refer to Pilz. The notion of bi-ideals in near-rings was effectively used to characterize the near-fields. Tamizh Chel-vam and Ganesan introduced the notion of bi-ideals in near-rings. Further Tamizh Chelvam introduced the concept of b-regular near-rings and obtained equivalent conditions for regularity in terms of bi-ideals. In fact, the result of Chelvam in that context generalizes the result of Kovacs for rings.

2. Cohomology of Semigroups

M. Loganathan
Professor, Ramanujan Institute of Mathematics,
University of Madras, Chennai-600 005

In this talk we discuss properties of various Cohomology theories that are associated with a semigroup.

3. Cross-Connections

K.S.S. Nambooripad
Former Professor, Department of Mathematics,
University of Kerala

The concept of *cross-connections* between partially ordered sets was originally introduced by P.A. Grillet in order to classify the class of fundamental regular semigroups. Grillet characterised the partially ordered set of principal left ideals and right ideals of regular semigroups as regular partially ordered sets and introduced an appropriate concept of duality for them. According to Grillet a cross-connection between two regular partially ordered sets *I* and *Ë* is a pair of order-preserving mappings

Ã : *I*→Ë* and Ä : Ë→ *I**

satisfying certain axioms, where *I** denote the dual of *I*. Every regular semigroup *S* induces a cross-connection between a partially ordered set *Ë(S)* of all principal left ideals of *S* under inclusion and the partially ordered set *I(S)* of all principal right ideals. Here this is generalised to cross-connections between normal categories. Again, every regular semigroup *S* induces a cross-connection between categoris *L(S)* and *R(S)* of principal left and right ideals.

We observe that the configuration that occur in Grillet's definition as well as its generalisation given here occurs in many areas in mathematics.

4. Limits over Categories of Extensions

Inder Bir S. Passi

Indian Institute of Science Education and Research,
Mohali and Panjab University, Chandigarh

Abstract

We will discuss limits over categories of extensions and explain how certain well-known functions on the category of groups, including group homology, turn out as such limits.

5. Normal Categories and Groupoids

A.R. Rajan

Department of Mathematics, University of Kerala

Normal Categories were introduced by K.S.S. Nambooripad towards providing general structure theorem for regular semigroups. The morphisms in this category admit a well structured factorisation in which one factor is an isomorphism. These isomorphisms are separated to form a groupoid which is a subcategory of the original normal category. Two more specified subcategories are identified and the normal category is described in terms of these subcategories.

6. Irreducible *p, q*-representations of the Lie Algebra *gl* (2) and *p, q*-Mellin Integral Transformation

Vivek Sahai* & Sarasvati Yadav

Department of Mathematics & Astronomy
Lucknow University, Lucknow.
E-mail: sahai_vivek@hotmail.com

Abstract

The theory of irreducible representations of Lie algebras together with the theory of integral transformations has been a rich source of results in special function theory. In particular, the Mellin integral transformation as well as its *q*-analogue have been studied from the Lie algebraic point of view resulting in special function identities and recurrence relations. In the proposed talk, we discuss new models of the irreducible *p, q*-representations of the Lie algebra *gl* (2). We introduce *p, q*-Mellin integral transformation and use it to transform the models of *gl* (2) and obtain identities involving *p, q*-special functions.

Symposium - 2

RECENT TRENDS IN DISCRETE MATHEMATICS

Organiser : Prof. A.K. Agarwal

1. Gaussian Binomial Coefficients in Geometry and Coding Theory

S. Ghorpade

IIT, Mumbai

ABSTRACT

Gaussian binomial coefficients go back to Gauss and are a natural generalization of binomial coefficients. Indeed, just as the binomial coefficients correspond to the number of subsets of a fixed cardinality of a given finite set, the Gaussian binomial coefficients correspond to the number of subspaces of a fixed dimension of a given finite dimensional vector space over a finite field. The Gaussian binomial coefficients are polynomials in the parameter corresponding to the number of elements of the relevant finite field. Moreover, their coefficients are nonnegative integers and admit a variety of equivalent descriptions. As such, these polynomials have been of considerable interest in combinatorics, especially the theory of partitions. In this talk I will first explain how Gaussian binomial coefficients arise naturally in algebraic geometry and topology in the study of Grassmann varieties or more generally, partial flag varieties. In particular, we will highlight the fact that Gaussian binomial coefficients neatly illustrate the celebrated conjectures of A. Weil. Next, we will consider the connection of Gaussian binomial coefficients with coding theory. Finally, we will outline some recent work on linear error correcting codes associated to Grassmann varieties wherein Gaussian binomial coefficients play a useful role.

2. Pseudo Rank Functions on Bounded Lattices

S. K. Nimbhorkar
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ABSTRACT

Dimension functions on lattices are introduced by von Neumann. These have been studied in various branches of mathematics including Operator Theory and Module theory.

In this talk a brief survey of dimension functions and its generalizations is given. In particular, the concept of a pseudo-rank function on a bounded lattice is introduced and its various properties are obtained.

It is shown that if L is a complemented modular lattice of finite length, then there exists a pseudo-rank function on L. Moreover, it is proved that if L is a relatively complemented lattice, such that the set of pseudo-rank functions is nonempty, then this set is a Choquet simplex.

3. Identities of the Rogers-Ramanujan Type

Pranjal Rajkhowa & F.A. Ahmed
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Abstract

We derive a general result for Rogers-Ramanujan Type Identities modulo (2p+1)s, where p is any integer greater than or equal to 3, and s is any natural number. Particular cases for Identities modulo 29 and 31 are deduced.

We also give the combinatorial interpretation of these and some other such identities.

4. Split-off Operation for Graphs, Matroids and its Applications

M. M. Shikare

University of Pune

Abstract

Let G be a graph. Given two adjacent non loop edges $x = vv_1$ and $y = vv_2$ in G, we construct a new graph G_{xy} by adding the edge v_1v_2 and deleting the edges x and y. The transition from G to G_{xy} is called a split-off operation.

The split-off operation is a well-known and useful method for solving problems in graph connectivity. This operation may decrease the edge-connectivity of the graph. We discuss various applications of this operation in graph theory. The split-off operation can be extended from graphs to binary matroids. In the context of matroids, we consider some applications of this operation.

5. Graph Operators and its Dynamics

Ambat Vijayakumar
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Cochin University of Science and Technology
Cochin-682 022 India
vambat@gmail.com

Abstract

In this talk we shall discuss graph operators, properties of its iterations and its dynamics. The operators of concern will be Line graphs, Gallai graphs, Anti Gallai graphs, Median graphs and P3 intersection graphs. This will also be a survey of our work in collaboration with S. B. Rao, Aparna Lakshmanan and Manju K. Menon. Some open problems will also be mentioned.

6. On Operations Preserving Some Connectedness of Matroids

B. N. Waphare
University of Pune

Abstract

Using basic operations like element addition and series extension, we characterized connected matorids. A theory binary matroids is developed using splitting operation and its variations.

Symposium - 3
MATHEMATICS AND STATISTICS - AN INTERDISCIPLINARY APPROACH

Organiser : Prof. A.M. Mathai

1. Distribution of Ratio of Extreme of Isotonic Estimators of Chi-Square Variables with Applications

Amar Nath Gill

Department of Statistics
Panjab University, Chandigarh

Key words: **Isotonic estimator; Scale parameter; Simple ordered alternative; Critical constants; Recursive integration; Constant failure rate; Normalized spacing between successive order statistics; Increasing failure.**

Abstract

Let $V_1,...,V_k$ be k independent chi-square variables each with v degrees of freedom (d.f.) and let $W_1 \leq ... \leq W_k$ be the corresponding isotonic estimators computed by the pool adjacent violator algorithm under simple ordering among Vs. In this paper, a recursive integration algorithm for computing the distribution function of statistic $T=W_k/W_1$ is presented. A test statistic $F^*_{\max}$ proposed by Fujino (Biometrika 66 (1979) 133-139) to test the homogeneity of variances of k normal populations against simple ordered alternative follows same null distribution as of the statistic T. Fujino discussed the null distribution of $F^*_{\max}$ and provided critical constant only for k=3,...,6, which limited its applications. The recursive procedure given in this article can be implemented efficiently for sufficient higher value of k. Test procedures based on statistic T for testing homogeneity against simple ordered alternative of k normal variances and equality of scale parameters of k exponential distributions are discussed separately with tables of critical constants for their implementation. Correctness of the critical constants is verified through simulation study. The proposed

methodology is also extended to address the problem of testing exponentially (constant failure rate) versus increasing failure rate, using isotonic estimators of normalized spaces between consecutive order statistics, along with tables of relevant critical constants.

2. Exponentited Weibull Distributions

K. Jayakumar
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University of Calicut, Kerala-673 635, INDIA

Key Words**: Discrimination Study, Exponentiated Exponential, Gamma, RML Method.**

In reliability studies one important distribution with positive support is the Weibull family of distributions. This family of distributions has found to be of considerable interest in modeling situations with monotone failure rates. In real practice, many situations arise which cannot be modeled by the members of failure models of this family and other family of distributions. The statisticians have been searching new models for modeling data that often exhibit non monotone failure rates using some improvement for this distribution. In the present work, basic properties of EW distribution are discussed. A process with EW marginal is introduced. Discrimination study between EW and gamma distribution is done and asymptotic properties of the discrimination statistic are derived. Data analysis for selecting appropriate model between Gamma and EW is carried out.

3. Statistical Distributions and Their Applications in Stochastic Process, Time Series Modeling, Reliability Analysis and Information Theory

K. K. Jose

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Statistical models are widely used in many areas like stochastic process modeling, reliability analysis etc. Various distributions like α-Laplace, asymmetric Laplace, Weibull, Mittag-Leffler etc have recently been developed by statisticians for modeling data sets from different contexts. We apply these distributions to develop new stochastic processes as well as time series models. The extended versions of some of these distributions corresponding to the pathway model give wide spread applications in reliability analysis, communicative engineering and statistical mechanics. The pathway parameter provides a transition to original form when it tends to unity. These distributions can have proper entropy interpretations, which are useful to develop the corresponding theory in statistical mechanics and communications engineering. The q--Weibull, Marshall-Olkin q-Weibull, q-Laplace and asymmetric q-Laplace distributions are developed and their applicability are extensively studied in various areas. A new time series model called max-min process is also developed.

4. Some Properties of Mittag-Leffler Functions and Matrix-Variate Analogues: A Statistical Perspective

A.M. Mathai

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Arunapuram, P.O. Kerala-686 574

Key words and Phrases: **Mittag-Leffler functions, Lévy density, Linnik density, Mellin-Barnes integrals, Multi-variate, Matrix-variate distributions.**

Abstract

Mittag-Leffler functions and their generalizations appear in a large variety of problems in different areas. When we move from total differential equations to fractional equations Mittag-Leffler functions come in naturally. Fractional reaction-diffusion problems in physical sciences and general input-output models in other disciplines are some of the examples in this direction. Some basic properties of Mittag-Leffler functions are examined first. Then representations in terms of Mellin-Barnes integrals are given, which are shown to yield many known and new results directly and easily. The results are presented in terms of statistical densities so that they are directly applicable to statistical distribution theory and stochastic processes. Several pathways are examined of exponential and gamma densities going to Mittag-Leffler densities and then Mittag-Leffler densities going to Lévy and Linnik densities. Then multivariable and matrix variable extensions of several results are given. Various results and representations given in this paper are directly applicable in many practical situations and are very suitable for further development of the theory. The material is presented in easily understandable formats, even for a beginner.

5. The Residual Effect of a Growth-Decay Mechanism and Fractional Calculus

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Key words: **Generalized Laplacian, Mittag-Leffler difference, Fractional integrals, Frac-tional derivatives.**

Abstract

There are various situations in which residual effect of an input activity and an output activity is observed. The difference of two independent gamma variables is shown to belong to a class called generalized Laplacian. A growth-decay mechanism is also shown to produce such a generalized Laplacian. Here we consider the Mittag--Leffler difference, which will leads to another class of generalized Laplacian model, say type-2 generalized Laplacian. We also discuss some properties of this new model and its relevance to time series. In this paper we establish connection of fractional integral operators to statistical distribution theory and incomplete integrals. Finally some applications of the above results are also listed.

6. The Statistical Mathematics of Some Combinatorial Identities

P. Yageen Thomas

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Kariavattom, Trivandrum-695 581

In this paper we describe the recently developed method of obtaining U-statistics based on "best linear unbiased estimators of location and scale parameter of a distribution by order statistics" as kernels. We utilize the results obtained in this case to generate a wide class of combinatorial identities. The exact forms of the identities corresponding to the U-statistics arising from some specific distributions are illustrated.

4. OTHER INVITED LECTURES

1. On a Fundamental Theorem of Fundamental Equations of Mathematical Physics

H. L. Manocha

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ABSTRACT

It has been observed that each fundamental equation of mathematical physics is featured by the fact that it has a solution space, which is generally a linear space in terms of hyper geometric functions, and at the same time it gives rise to a symmetry algebra, which turns out to be a Lie algebra. This rare characteristic suggests a connection between the two outcomes, leading to a theorem, aptly called a fundamental theorem associated with the fundamental

equations of mathematical physics. In consideration of this, a wave equation, which is one of the fundamental equations of mathematical physics, is cited and a technique is developed to find its solution space as well as a symmetry algebra associated with it and a few others. This leads to a theorem in terms of a representation theory of Lie algebras. This theorem is shown to contain ideas which have far reaching applications.

2. Applications of Lambert W-function

P.N. Rathie
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Key Words: **Lambert W-function, Mathematics, Statistical Distributions, Reliability Theory, Hydraulics, Information Theory, Electrical Engineering**

ABSTRACT

The Lambert W-function *W(x)* is defined as the solution of the equation $W(x)e^{w(x)} = x, x$å$[-1/e, \alpha)$. The W-function is recently used in various areas of mathematics, statistics, and engineering giving exact results. In some cases, it helps in reducing the number of parameters to be estimated. Computation of W-function, with good precision, is already available in Mathematica and Maple softwares for numerical applications.

The aim of this lecture is to show the importance of W-function in the solution of various new problems arising in

(a) Statistics (generalized gamma distribution, quantile function for some distributions, distribution of maximum and minimum of order statistics etc)
(b) Mathematics (solution of equations)
(c) Reliability Theory (optimal maintenance time for repairable systems)
(d) Hydraulics (flow problems in canals)
(e) Information Theory (Renyi entropy)
(f) Electrical Engineering (generalized Gaussian noise)

A few interesting known problems with their solutions in terms of W-

function may also be mentioned to emphasize the importance of this newly developed special function.

3. Bicomplex Analysis: New Horizons

Rajiv K. Srivastava

Prof. and Head, Department of Mathematics,
Institute of Basic Science, Dr. B. R. Ambedkar University,
KHANDARI Campus, AGRA - 282 002, (U.P.), INDIA

During the past decade, the theory of Bicomplex Numbers and Bicomplex Analysis have gained immense attention and importance amongst the contemporary workers due to its multidimensionality blended with commutativity - one of the most essential part of most of the physical systems.

Several new horizons have been touched by workers in this area. This includes Bicomplex Holomorphic Continuation, Tetralinear Transformations, Bicomplex Mirrors, Bicomplex Arguments, Bicomplex Beta and Gamma Functions, Bicomplex Riemann Zeta Functions and Spaces of Bicomplex Sequences and Series.

This talk is an attempt to present an almost updated account of work done in these new areas mainly by our group of workers.

4. Combinatorics on Finite Sets

N.K. Thakare
Dhule/Pune

Abstract

In 1928, E. Sperner stated the maximum number of subsets of a finite set subject to the condition that no subset is contained in another subset. This theorem has been reproved and generalized leading to a new theory called "Combinatorics On Finite Partially Ordered Sets" (also called Sperner Theory On Finite Posets).

In this expository talk we shall outline a few proofs of original theorem as well as give its generalizations to multisets.

A few open problems will be discussed.

Sample Problem:

Is the poset L(m,n), (m,n positive integers), Symmetric Chain Order ?

Related Enumerative problems such as the number of lattices with *n* elements, the number of antichains of a finite set will also be touched upon.

5. Dimension Functions on Rings and Their Generalizations

M. P. Wasadikar
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Dr. B. A. M. University Aurangabad-431 004

Abstract

Von Neumann introduced the concept of a dimension function in continuous

geometry. Many researchers like Kaplansky, Maeda and Loomis studied dimension theory of structures such as Banach Algebras, Lattices and Operator algebras. Goodearl and Boyle extended this theory to non-singular injective modules. Berberian studied dimension theory in Baer *-rings. Dimension functions are generalized to rank functions and pseudo-rank functions by Goodearl.

In the proposed talk, we give a brief survey of dimension functions and pseudo-rank functions in various structures and in particular in Rickart *-rings. We show how new pseudo-rank functions can be constructed from the given ones. The existence of pseudo-rank functions on certain Rickart *-rings is shown. Some other properties of such functions are proved.

6. Certain Type of Modular Sequence Spaces

Manjul Gupta

Department of Mathematics & Statistics
Indian Institute of Technology
Kanpur-208016, India

Abstract

In this talk we shall consider the structural properties of particular type of modular sequence spaces defined with the help of a given sequence $á=\{ á_n\}$ of strictly positive real numbers $á_n$'s and an Orlicz function *M*. These spaces, in particular include the sequential analogues of certain subspaces of the space of entire functions.

7. Number Theory Challenges of 21^{st} Century

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Abstract

Problems in number theory are sometimes easy to state and often very hard to solve. We survey some of them.

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V

ABSTRACTS OF

Oral/ Poster Presentations

1.1 MATHEMATICS - ORAL PRESENTATIONS

1. A New Class of Lattice Paths and Partitions with n Copies of n

S. Anand & A. K. Agarwal
Dept. of Mathematics
Panjab University, Chandigarh

Key Words: **Partitions, Compositions, Lattice Paths, Partitions with n copies of n, Graphs, Combinatorial identities.**

Abstract

In [A. K. Agarwal and D. M. Bressoud, Lattice paths and multiple basic hyper geometric series, Pacific J. Math., 136, No. 2 (1982), 209-228] Agarwal and Bressoud defined a class of weighted lattice paths and interpreted several q-series combinatorially. Using the same class of lattice paths, Agarwal in [A. K. Agarwal, Lattice paths and *n*-color partition, Utilitas Math., 53(1998), 71-80; A. K. Agarwal, New classes of combinatorial identities, ARS Combinatoria, 76 (2005), 151-160] provided combinatorial interpretations for several more q-series. In this paper, a new class of weighted lattice paths, which we call associated lattice paths, is introduced. It is shown that these new lattice paths can also be used for giving combinatorial meaning to certain q-series. However, the main advantage of our associated lattice paths is: they provide a graphical representation for partitions with $n+t$ copies of n introduced and studied by Agarwal in [A. K. Agarwal, Partitions with *n* copies of *n*, Lecture Notes in Math., No. 1231, Springer- Verlag, Berlin/ NewYork, (1985), 1- 4.] and Agarwal and Andrews in [A. K. Agarwal and G. E. Andrews, Rogers-Ramanujan Identities for partitions with *n* copies of *n*, J. Combin. Theory Ser. A 45, No. 1(1987), 40- 49].

2. Analytical Approach for High Eccentricity Satellite Orbits with Atmospheric Drag using KS Uniformly Regular Canonical Elements

C.S. Anitha# & R. K. Sharma*

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**. Vikram Sarabhai Space Centre, Applied Mathematics Division, Thiruvananthapuram-695 022, India*

Key words: **Non-singular analytical theory, High eccentricity, Air drag, KS uniformly regular canonical elements, Spherically symmetrical atmosphere**

Abstract

A non-singular analytical theory for the contraction of high eccentricity satellite orbits with air drag effect is developed in terms of the KS uniformly regular canonical elements, using a spherically symmetrical atmospheric model. The series expansions up to sixth power in terms of an independent variable ë, introduced by Sterne as cos E= 1-H ë^2/ae, where E, a and e being the eccentric anomaly, semi-major axis and eccentricity, respectively. The density scale height H is assumed as constant. Only two of the nine equations are solved analytically to compute the stage vector and change in energy at the end of each revolution, due to symmetry in the equations of motion. Comparison of the important orbital parameters semi-major axis and eccentricity up to 1000 revolutions, obtained with the present analytical solution, with KS elements analytical solution and with respect to numerical integration, shows the superiority of the present solution over the KS elements solution. The theory can be used effectively for the orbital decay of aero-assisted orbital transfer orbits during mission planning.

3. Periodic Orbits in the Restricted Three-Body Problem using Poincare Surface of Section Method

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Key words: **Poincare surface of section; quasi-periodic orbits; periodic orbits; oblateness.**

Abstract

The paper investigates the family of periodic orbits in the rotating Earth-Moon system in the planar circular restricted three-body problem using the numerical technique of Poincare surface of section. It is further concerned with generating periodic orbits, by considering the centre of regular regions in the surface of sections. The effect of the Earth's oblateness on these periodic orbits is studied. Due to Earth's oblateness, the period of short-period orbits around the Earth decreases, whereas those with long-period increases. In the case of the periodic orbits around the Moon, the period of the orbits starting with x< Lagrangian point L_1 (0.83689) increases, whereas those starting with x> L_1 decreases due to the Earth's oblateness.

4. Evolution of Vortex Knots

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Key words: **Vorticity Curvature, Torsion, Frenet-Serret Equations.**

ABSTRACT

The dynamical behaviour of a closed vortex filament is a complicated problem. A perturbation method introduced by Keener [Knotted vortex filaments in an ideal fluid, J. Fluid Mech. 211, 1990] gives new opening for dealing with this problem. Here a knotted curve is a realized as bifurcating from a circle. The basic principle is that a circle is an unknot and other knots are obtained starting from a circle in the reverse order.

Though Keener calls his knots as knotted vortex filaments, he deals only with the geometry of a closed curve. Hence this method has the drawback that it does not take into account the stretching of vortex lines or filament, which is the key-hydrodynamic contribution.

As in keener, we also find that there are torus knots that are obtained as solution of the Frenet-Serret Equation. These can have only arbitrary winding number and move as rigid bodies. The invariant solutions are of solitary structure as the equation of motion for curvature and tension can be transformed into nonlinear Shrondinger equation. For each rational number m/n there are left handed there are left handed and right handed torus knots that rotate about their own axis of symmetry in opposite directions.

5. Discrete Time Queueing Models of Computer Communication Network

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Key Words : **Discrete time models, Slotted network, Packet data**

ABSTRACT

In this paper, we analyze discrete time queuing models for computer communication network. In a discrete time model time is assumed to be "slotted". A discrete time queue might accept at most one packet during a slot and service at most one packet during slot. This paper concludes with three case studies involving high-speed networks. This includes a study of a canonical discrete time arrival process. In the more modern packet switching technology, packets of bits pass through the inter connection network.

6. A Survey on Development in Sequence Spaces and their Duals and Bicomplex Kothe - Toeplitz Duals.

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Key Words: **Köthe-Toeplitz dual, Generalized Köthe-Toeplitz dual, Bicomplex Köthe--Toeplitz dual, Difference sequence space**

Abstract

A survey on recent developments in the duality theory of sequence spaces has been done and Bicomplex Köthe - Toeplitz duals have been defined. Sixteen types of different duals have been given and relation between them is studied.

7. The Dirac Equation : An Approach Through Geometric Algebra

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Cochin, Kerala, India.

Key words: **Geometric algebra, multi vector, Clifford algebra, spinor, space time, Dirac equation, Klein-Gordon equation, Zitterbewegung.**

Abstract

We solve the Dirac equation in a manner similar to Toyoki Koga, but using the geometric theory of Clifford algebras initiated by David Hestenes. Our solution exhibits a spinning field, among other things.

8. Frobenius Partitions and Gordon-Gollnitz Identities

M. Rana & A.K. Agarwal

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Key words: **Coloured partition, Frobenius partition, lattice paths, Weighted difference.**

Abstract

Using Frobenius partitions we extend our main results of A. K. Agarwal and M. Rana, "New Combinatorial versions of Göllnitz-Gordon identities," Utilitas Mathematics, 79(2009), PP. 145-155. This leads to an infinite family of 4-way combinatorial identities. In some particular cases we get even 5-way combinatorial identities, which give us four new combinatorial versions of Göllnitz-Gordon identities.

9. On Certain Bicomplex Order Topologies

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Key words: **Bicomplex Numbers, Dictionary Order, Order Topology, Product Topology, Metric Topology.**

Abstract

In this paper, we have defined two types of order relation in the bicomplex space with the help of dictionary order relation in the complex space. We have proved that these two types of order relation in the bicomplex space are

independent of each other. With the help of these orderings in the bicomplex space we have defined two order topologies, two product topologies and two metric topologies on the bicomplex space. Further we have compared these topologies on the bicomplex space. We also have compared our results with results of the other papers [S1] and [S2].

10. On Zeros and Euler Product Representation of Generalized Bicomplex Riemann Zeta Function

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Key words: **Bicomplex Numbers, Dirichlet Series, Euler Product**

Abstract

In this work, we have used Bicomplex numbers (cf. **[P1], [S1]** and **[S2]**) introduced the Bicomplex version of ordinary complex Dirichlet series which we call as Generalized Bicomplex Riemann Zeta Function. Furthermore, we have obtained a zero free region of Generalized Bicomplex Riemann Zeta function in a case when Generalized Bicomplex Riemann Zeta function becomes a Power series and we obtain a bicomplex Euler product of Generalized Bicomplex Riemann Zeta Function.

11. Topological Invariants in Hydrodynamics

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Cochin University of Science and Technology,
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Abstract

A criterion using Lie derivative is used to obtain invariants in hydrodynamics by considering a four dimensional space time manifold E^4. We define a closed differential two form and its potential one form in E^4. Using this a three form and a four form are defined and invariance of these forms are discussed. These are explained using some examples.

12. Propagation of Lamb Waves in Porous Piezoelectric Plate

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Key words: **Lamb wave, Wave propagation, Dispersion, Porous piezoelectric, PZT**

Abstract

The propagation of symmetric and anti-symmetric Lamb wave modes in porous piezoelectric plate, having 6mm crystal symmetry, is studied analytically. The Christoffel equation of plane harmonic waves propagating in porous piezoelectric solid is derived in term of slowness. The dispersion relation for the symmetric and anti-symmetric Lamb wave modes are obtained by separating the closed form solution into symmetric and anti--symmetric

modes. The effect of different electrical boundary conditions i.e. shorted case, free case and shorted-free case on the characteristics of lamb waves is studied. The transcendental dispersion relation is solved numerically for a particular model PZT. The effects of frequency, plate thickness, direction of propagation and the porosity on the phase velocities of Lamb waves are studied.

1.2 MATHEMATICS - POSTER PRESENTATIONS

13. Bilateral Mock Theta Functions of Order "Seventeen"

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Key words: **Mock theta functions, Bilateral mock theta functions, Hyper geometric series, Characteristic property, Unit circle 33 D.**

Abstract

In this paper eight bilateral mock theta functions of order "Seventeen" were obtained using transformation theory of bilateral basic hyper geometric series. It has been shown that these functions are related to basic hyper geometric series ${}_9\phi_8$ and they satisfy the characteristic property of the mock theta functions defined by Ramanujan.

14. Calculation of Life Expectancy for Single Years-An Application of Numerical Analysis

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Abstract

Abridged life tables are based on the assumption that the mortality rates across five year age group remains the same although this assumption may not be valid in many instances particularly in both ends of the age distribution. In recent years, with the fast change in adult mortality, the single year age distribution is of much importance although due to want of data, most of the life tables invariably confine to five year age groups.

Here an attempt has been made to develop a Mathematical model and its empirical application of converting the abridged life expectancy into single year life expectancy figures. That is the life expectancy figures for each age without the information on the single year age specific death rate.

This method helps to identify and understand the relative contribution of life expectancy in different ages. The application of the methods to India and fifteen states has shown that life expectancy at each age 0,1,2,3 to 70 is obtainable. Usually the levels of life expectancy increases immediately after age zero and after a few years shows the declining trend. Here in this study, for Andhra Pradesh the life expectancy at age zero is 65.5 years, a slight increase after that and the declines start only at age five. Similarly in Madhya Pradesh, the decline start only after age 10. Hence this methodology is useful in the decomposition of life expectancy into single years.

15. A Note on Fixed Point Property in Posets

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Key words: **Poset, Fixed point property, Isometric set, Retract, Chain-completeness.**

ABSTRACT

In this paper, some of the well known results for isometric fences for posets are refined. Further, a necessary and sufficient condition for the set of all external elements of a poset to be a retract is given by the exclusion of a particular type of subposet. In the absence of this type of subposet, the comparable point property is characterized for posets. This in particular, characterizes the fixed point property to all chain--complete posets upto the exclusion of the particular subposet.

16. On Generalized Nörlund Summability Factors of Infinite Series

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ABSTRACT

In this paper I have proved a theorem on generalized Nörlund summability factors of infinite series, which generalizes various known results.

17. Propagation of Magnetoelastic Shear Wave in Multilayered Anisotropic Media

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Key words: **SH waves, Magnetoelastic, Self-reinforcement, Seismic wave, Dispersion equation.**

Abstract

The present paper reports the study of the propagation of horizontally polarised magnetoelastic shear harmonic waves in a self-reinforced (anisotropic) media. The medium consist of (n-1) layers lying over a half space. A general dispersion relation has been obtained analytically using Haskell's matrix method. It has been observed that the obtained dispersion relation reduced to the classical SH wave equation for the isotropic case when a single or double layer is considered over a half space. It is distinctly marked that phase velocity dispersion curve is affected by magnetoelastic self-reinforced parameter.

18. All things are Numbers – A Pythagorean Idea

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(Accredited A+ by NAAC, CPE by UGC)

Abstract

The philosophy of numbers that Pythagoras is associated with incorporates mathematical laws and geometric figures as proof that numbers are fundamental elements of the universe. To Pythagoreans, numbers are more than abstract figures. The Pythagoreans saw mathematics and geometry as sacred tools for uncovering the true nature of the universe. The contrarieties odd and even square and oblong triangular numbers etc; are related to the Pythagorean representation of numbers as patterns of dots.

The present paper tries to relate his philosophy of numbers, transmigration of soul and cosmos with properties of numbers.

19. Effects of the Perturbative Forces on the Motion and Stability of Inter-Connected Satellites System in Orbit

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Key words: **Inter-connected Satellites, Air drag, Magnetic field, Stability.**

ABSTRACT

The paper aims at studying the effects of air drag and pull of terrestrial magnetic field on the non-linear motion and stability of two cable - connected

artificial Earth Satellites system in the central gravitational field of the Earth. The equations of motion of the system have been deduced with respect to the centre of mass of the system, which is assumed to move along a Keplerian elliptic orbit (in particular circular). Hence a set of non-linear, non-homogeneous and non-autonomous differential equations in rotating frame of reference as well as Nechvile's co-ordinates has been obtained for the motion of the system.

The 'Jacobi' integrals exists for the problem and hence with the help of this integral, a sufficient conditions for the stability of the system is deduced.

20. On Momentum of Progressive Surface Waves of Small Amplitude : A New Physical Significance of the Wave Velocity

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Abstract

The momentum content per wave length as well as per unit length in progressive waves of small amplitude on the surface of water of arbitrary depth is calculated, and the variation of momentum with the wave length, on water of different depths, is investigated. The velocity of momentum transmission in the waves is also calculated; this velocity turns out to be equal to the velocity of wave propagation and furnishes, thereby, a new physical significance to the wave velocity of progressive surface waves.

21. π* On Squaring the Circle

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Abstract

There is a fundamental problem of mathematics "squaring the circle" in world that how to draw a circle with equal area of square. The above problem has been created only, as Pi is irrational, rather a transcendental. Since pi is a numerical constant express the relationship between the circumference of the circle and its diameter on a flat plane surface. So with the help of series of experiment a number which is quite closer to the approximated value of pi is determined. The experiments were performed Buffon's needle probability theory, volume of sphere, squaring the circle etc. and get a number 3.24 denoted by π*.

22. Drag on a Fluid Sphere Embedded in a Porous Medium

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Key Words**: Brinkman equation, Modified Bessel functions, Drag force.**

Abstract

The problem of Stokes flow past a fluid sphere embedded in a porous

medium is studied. The Brinkman equation for the flow outside the fluid sphere and Stokes equation inside the fluid sphere, in their stream function formulation are used. The drag force experienced by a fluid sphere embedded in a porous medium is evaluated. The dependence of the drag coefficient on permeability and viscosity ratio is presented graphically and analytically. Some previous known results are also deduced from the present analysis.

23. *GαO-Kernel in the Digital Plane

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Abstract

Digital topology was first studied in the late 1960's by the computer image analysis researcher Azriel Rosenfeld. The digital plane is a mathematical model of the computer screen. In this paper we investigate explicit forms of *GαO-kernel and *gα-closed sets in the digital plane.

24. Fear: A Mathematical Model

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Key Words: **Fear, Generalized information about the possible threat, Incoming information about the anticipated or real threat, Fear coefficient.**

Abstract

Fear is an emotional response to danger and threats. The feeling of fear may generate due to something or somebody. In the present work we developed a mathematical model of fear taking into account the generalized information about the possible threat stored in the memory and the incoming information about the real (or anticipated) threat. The possible trend of the coefficient of fear is attempted to be traced in the present analysis. In our present model we have also analysed the case if there appears any external factor and starts its action to remove the fear.

25. Axisymmetric Creeping Flow of a Micro Polar Fluid Over a Sphere Coated with a Thin Fluid Film

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Key words: **Micro polar fluid; Stokes flow; Modified Bessel functions; Drag force.**

Abstract

This paper concerns the problem of steady axisymmetric Stokes flow of a micro polar fluid past a sphere coated with a thin, immiscible Newtonian fluid layer. Inertial effects are neglected for both the outer fluid and the thin fluid film. The stream function solution is obtained in terms of modified Bessel and Gegenbauer functions. The constants appearing in the flow field are computed by matching the boundary conditions at the coated sphere interface. The inner and outer fluid motions have been determined. The drag force experienced by a fluid-coated sphere is evaluated, and its variation is studied on its geometric and material parameters. Some well-known results are then deduced.

26. Role of Super Symmetry in C^3-Space

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Key words: **Supersymmetry, C^3-Space, Bosons, Fermions, Bradyons, Tachyons, Pseudo-Lie algebra.**

ABSTRACT

Bosons are particles of zero or integral spin obeying Bose-Einstein statistics. Fermions are half-integral spin particles that follow Pauli's exclusion principle and obey Fermi-Dirac statistics. Unlike Maxwell-Boltzmann statistics for the molecules of a gas these bosons and fermions undergo quantum statistics as in an ensemble they are not only identical but also indistinguishable. In the present work we have considered the C^3-space and dealt with how the super symmetry of bosons and fermions with bradyons and tachyons can be shown through a closed system of commutation and anticommutation relations under pseudo-Lie algebra.

27. On Divisibility and Squares

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Abstract

In this note we have derived a new method to find the square of any number and devised a unique methodology for divisibility of numbers 7 and 13. We have discussed the relevant theory and provided some illustrations in its support.

28. An Analytical Method for Computation of Prime Numbers

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Abstract

An Algorithm for generation of large Prime Numbers have been developed by the author. This method has the unique feature in the sense that it is capable of computing Prime Numbers, as long as practicable, by calculating the residues of mod (P), P being the Prime Number, utilizing the well-known Fermat Little Theorem for Prime Numbers.

29. Reformulation Techniques in Logic Based Optimization

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Key words: **Linear constraints, Non Convex, Branch and Bound.**

Abstract

This paper treats the solution of nonlinear optimization on problems involving discrete decision variable, also known as generalized disjunctive programming (GDP) or mixed integer nonlinear programming (MINLP) Problems. A novel automatic reformulation method for non convex NLPs that include Linear constraints and bilinear terms. Many engineering optimization Problems can be formulated as non-convex non--Linear Programming Problem involving a non-linear objective function subject to non--Linear constraint and concerned with techniques for establishing such global optimal using spatial Branch and Bound Algorithm.

30. Liquid-Metal Duct Flow Close to a Magnetic Neutral Point

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Key words**: Magneto hydrodynamics; Liquid-metal flows; Non-conducting duct; Unidirectional flows; Lorentz force.**

ABSTRACT

The steady-state two-dimensional flow structure of a unidirectional solution is examined for both the cases of a conducting liquid metal passing through a magnetic neutral point and that of a uniform transverse magnetic field

(Hartmann flow). Electromagnetic braking results from the action of Lorentz forces upon the metal when the flow impinges upon a region of d. c. magnetic field. Numerical studies indicate that the non-linear flow becomes unidirectional and linear near a local neutral point. This linear behaviour accords well with analytic solutions for flow through an infinitely extended magnetic neutral point. The latter forms one of a family of magnetic stream functions. We investigate that permits such unidirectional flow.

31. A Brief Study of Contributions of Nasir Al-Din-Al-Tusi to Mathematics and Astronomy

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Ashwini Kumar Sinha
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Abstract

In this paper we have attempted to take a view of unforgettable contribution of the all time great astronomer, mathematician and philosopher Nasir-Al-Din-Al-Tusi (1201-1274).

32. Solutions of Generalized Photogravitational Elliptic Restricted Three Body Problem

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Key words: **Solutions/Generalized/Photo gravitational/ERTBP**.

ABSTRACT

We attempted to find the solutions of generalized photo gravitational elliptic restricted three body problem. The problem is generalized in the sense that both primaries are supposed to be an oblate spheroid. By photo gravitational we mean that both primaries are radiating as well. We have found the particular solutions. They depend upon radiation and oblateness of primaries. Classical results may be verified from this result.

33. Studies on the Aryabhata-I's Methods and Modern Methods for Square-root

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Key words: **Square-root, Whole Number, Decimal Number.**

Abstract

The different steps of the method given by Aryabhata-l for extracting a square-root of a whole number and to extend the method for extracting the root of a decimal number. It is also explained by Bhaskara-l and other ancient scholars. The history of mankind shows different thoughts about human nature at their beginning on the earth. Putting the names of Hobbes; Roussace etc. for their thoughts about the states of nature in this line. As the history of mathematics is a history of mankind because it consists the intellectuality, of the human race or of a society for their development. The study of history of mathematics reveals the modern and ancient methods are comparatively quite useful. But it is found, for few cases that modern methods have not yet elaborated or generalized the method of extracting square-root of a whole number given by the Ancient Indian mathematics. The modern methods motivated much more from abstraction and generalization than from applications. This method is very popular for extracting the square-root of a number.

34. Mathematical Methods in Medical Image Processing

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Key words: **Medical imaging, artificial vision, partial differential equations, energy.**

Abstract

In this paper, we describe some central mathematical problems in medical imaging. The subject has been undergoing rapid changes driven by better hardware and software. Much of the software is based on novel methods utilizing; geometric partial differential equations in conjunction with standard signal/image processing techniques. As part of this enterprise, researchers have been trying to base biomedical engineering principles on rigorous mathematical foundations.

35. An Investigation into Effect of Electromagnetic Fields on Generalised Couette Flow with Heat Transfer

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KEY WORDS: Open Circuit, back flow, Hartmann number.

Abstract

One of the basic problem, the plane Couette flow, has been a source of many research workers in dealing with the interplay of various fluid forces

and their interaction with the electromagnetic forces. The effect of electromagnetic fields on (a) separation tendency in generalized Couette flow, and (b) heat transfer in generalized Couette flow has been investigated. For the physical insight of problem velocity distribution, temperature field is obtained and with the aid of it the local Nusselt number is derived.

36. A Fuzzy Production Inventory Model Over a Random Planning Horizon

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KEY WORDS: **Production inventory model; Random planning horizon; Genetic algorithm; Possibility.**

ABSTRACT

A production inventory model with stock dependent demand is developed where the production is made in production house in industrial area and the products are sold in the market place under two storage facilities over a random planning horizon, which is assumed to follow exponential distribution with known parameter under fuzzy space constraint. The fuzzy space constraint is transformed into deterministic one using possibility measure. The expected profit is derived and maximized with constraints via genetic algorithm (GA). The model is illustrated with some numerical data. Sensitivity analyzes on expected profit function with respect to confidence level 17 of space constraint are also presented.

37. Some Properties of Kenmotsu Manifold Admitting a Quarter Symmetric Non Metric Connection

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Key words: **Kenmotsu manifold, Lemmas and Theorems, Concircular Curvature Tensor, Conharmonic Curvature Tensor.**

Abstract

The purpose of this paper is to study some of the properties of quarter symmetric nonmetric connection in Kenmotsu manifold.

38. Mathematical Modeling for Arm Race Between Two Countries by Differential Equation

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Abstract

We want a mathematical model to be as realistic as possible and to represent reality as closely as possible. However, if a model is very realistic, it may not be mathematically tractable. In making Mathematical model, there has to be a trade-off between tractaleility and reality.

We know that Mathematical Modelling is not a one-shot affair. Models are constantly improved to make them more realistic. Thus for every situation, we get a hierarchy of models, each more realistic than the preceding and each likely to be followed by a better one.

Mathematical models can lead to new experiments, new concepts and new Mathematics. Comparison of predictions with observations reveals the need for experiments to collect needed data. It can also lead to development of new concepts. If known Mathematical techniques are not adequate to deduce results from the Mathematical model, new Mathematical techniques have to be developed.

Keeping the above principles in mind we have developed a Mathematical model for Arm Race between two Countries by Differential Equation.

39. Congruence Relations in Trellises

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Key words: **Trellis, Lattice, Ideal Lattice of a Trellis, Congruence Relation.**

Abstract

In this paper an attempt has been made to extend the ideal theory of lattices to trellises. Hashimoto's results concerning ideals and congruences in lattices are extended to trellises.

40. Modern Cosmological Ideas in Ancient India

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Key Words**: Cosmology, Cosmogony, Big-Bang, Protoplanet Hypothesis, Nebular Hypothesis, Cyclic Model, Heliocentric.**

Abstract

The modern concept of space - Time and Theory of Cyclic model, Nebular hypothesis, Protoplanet hypothesis, Cosmogony, Cosmic elements, Matter and Energy etc. also exist in Ancient Indian Sanskrit literature. We examine exclusively about the modern cosmological ideas which were found in Rigveda and other vedas, Satapatha Brahmana, Aitereya Brahmana, Vedanga Jyotisa, Yoga-Vasistha, Vayu Purana etc. In this paper we have proved that the modern cosmological model also existed in ancient India.

Notation: In this paper we have adopted notation normally used in computer literature which is quite handy and less space-consuming E.g., symbol for multiplication will be *.

41. New Model on the Development of Mathematical Reasoning

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ABSTRACT

Mathematics is a way to settle in the mind a habit of reasoning where the results are developed through a process of reasoning and the conclusions are drawn from the given facts. The reasoning in mathematics has the following characteristics such as simplicity, accuracy, certainty of results, originality, similarity to the reasoning of life, and verifications. Mathematics in the making is intuitive, experimental and inductive, where mathematicians generalize their theories from particular examples. Deductive reasoning proceeds from abstract rules to concrete ideas. mathematics, in its widest sense is the development of all types of deductive reasoning which requires undefined terms, definitions postulates, axioms and essentials. Study of mathematics disciplines the mind and develops the reasoning power of our different faculties. It gives us knowledge without taxing the memory in the least - resulting in the development of power rather than the acquisition of knowledge in short, model leads a student of mathematics to an intelligent and judicious mode of study in general and enables him to acquire mental self-reliance habits and independence in out-look.

42. On Lorentzian Almost Complex Manifolds

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Abstract

Lorentzian manifolds have been studied by several authors including Rossella Bartolo, Alfonso Romero and Miguel Sanchez. N.I. Nedic proved that a Lorentzian manifold is birecurrent if its isotropic base vectors are recurrent. He also discussed certain properties for such manifolds.

In this paper, we have studied the Lorentzian almost complex manifolds and obtained some results for such manifolds.

43. Optimal Ordering Polices to Inventory Model for Deteriorating Items having Pareto Distribution with Trapezoidal Type Demand and Backlogging

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Key Words: **Inventory, Trapezoidal demand rate, Backlogging, Pareto distribution.**

Abstract

In this paper, we study and analyze an inventory model with the assumption that the lifetime of the commodity is random and follows a generalized Pareto distribution. It is also assumed that demand rate is Trapezoidal type, i.e. the

demand rate is a piecewise linearly function and this demand rate is used when stock available as well as in shortage period. Shortage is completely backlogged. We proposed an optimal replenishment policy for this type of inventory model. With suitable cost consideration, the total cost function is obtained. Minimizing total cost function, the optimal ordering quantity and optimal time are obtained. The numerical example is presented to illustrate the model.

44. Rosenbrock Method for Optimization of Single-Variable Nonlinear Functions

Ashwini Kumar Sinha

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Alok Kumar

P.G. Deptt. of Maths, Maharaja College, ARA

Abstract

In this paper, a simple iterative procedure suggested by Rosenbrock to locate the optimum of a nonlinear function of a single variable has been discussed. The algorithm for finding the optimum has been described and the method has been illustrated by an example.

45. Study of Fuzzy Sets

Ashwani Kumar Sinha[1] & Alok Kumar[2]

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2. P.G. Deptt. of Mathematics, Maharaja College, Ara

Abstract

The Modern concept of naïve set theory was introduced by Cantor and Boole in the beginning of 20th century Let X be a universal set and A be a subset of X. Then A can be characterized by using its characteristic function.

In this definition, the belongingness of an element x in the subset A is either True or False. Black or White; Yes or No etc. This way of representation can only be imaginary; which cannot be realistic. Hence applying this concept to real problems will not give actual solutions. Starting from Russel in 1920 (Russel Paradox), Max Block in 1935. Mathematicians analyzed and understood this difficulty. But none could improve the concept. The improvement of the concept is actually the gradation of the belongingness of the element between 0 & 1, more or less between True or False, the Grey area between Black and White; the fuzz between a set and its complement.

Hence in 1965 cyberneticist Lotti A Zadeh in his seminal paper introduced fuzzy sets and applied to his control problems.

46. A Note on $|A,d,ß|_k$ Summabilty Factors of Infinite Series

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Abstract

$|A,ä,â|_k$ summable series is defined. Using this idea a theorem of Rhoades and Savas, Acta Maths Hunger. ,112 (1-2), 2006, 15-23 is generalized.

47. Calculation of Fraction of Last Year of Life- A New Approach (Life Table Techniques- A Comparison)

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Key words: **Life table, Chiang, Fraction of life**

Abstarct

Introduction:

The history of life table construction shows the successive refinement of the method. As the construction of life table procedure remained fix, but new methods to calculate its specific functions such as q_x and L_x have provided by researchers in the duration of times. One of the most useful methods, which widely utilized by WHO, is Chiang's method. Chiang introduced new formula for these two functions based on new concept, to show the relation between life table's functions.

Objectives

According to the Chiang's methods these formulas the most important element is fraction of life lived by dead persons. This element has been used to convert the age-specific death rate to the estimate of probability of death. The main objectives of this paper are explaining Chiang's method in finding the fractions and introducing a new approach and finally comparison of these two methods.

Method

The second method to calculate these fractions for every width age intervals is based on the concept of proportion of life which has lived by deaths. This method needs mortality data by part of a year; like days if applicable, months or seasons to get rid of fractions for single years of life based on Chiang's method.

Results

Chiang and new approach were used for mortality data to find out the fractions and after using statistical tests it is found that, there is significant difference. And then coefficient variation was used to find the variation for fractions which calculated by two methods and we got result, the Coefficient Variation for new approach is smaller.

48. Pulsatile Flow of a Casson Fluid Through a Stenotic Artery with Slip at Wall

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Key words: **Pulsatile flow; Casson fluid; Stenosis.**

Abstract

This paper deals with the pulsatile flow of casson fluid through a stenosed artery by considering the classical slip boundary condition at the wall. Analytical solutions are obtained using perturbation analysis. The approximations have been made on the assumption that Womersley number is very small making the solution suitable for smaller arteries. Expressions for velocity, plug core radius, plug core velocity, wall shear stress, slip velocity, flow rate and vascular impedance are derived and expressions for special cases are deduced. Effect of slip parameter on different hydrodynamic quantities is analyzed. Blood is taken as Casson fluid and the artery is not of uniform radius due to development of stenosis at several places. Particular cases of Newtonian fluid and some previously established results have been verified.

49. On Degree of Approximation of Function Belonging to the Lipschitz Class by Almost (C,1) (E,1) Means of its Fourier Series

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Abstract

In this paper, a theorem on the degree of approximation of the function belonging to the Lipschitz class by almost (C,1)(E,1) product means of its Fourier series has been established.

2.1 STATISTICS - ORAL PRESENTATIONS

50. Gamma Density and Extensions by using Pathway Idea

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Key words: **Pathway model, Extended gamma, Krätzel integral, Reaction rate inte-gral, *H*- function, Super statistics**

Abstract

By taking the pathway model of Mathai (2005, Linear Algebra and its Applications, 396, 317-328) as a basis, Laplace transforms of generalized and extended forms of gamma densities are evaluated. These Laplace transforms provide connections to inverse Gaussian density in statistics, Krätzel transforms in applied analysis, moment generating functions, reaction rate probability integrals in astrophysics, Tsallis statistics and super statistics in non-extensive statistical mechanics, etc. Multivariate analogues are also examined, which will provide multivariate extensions to all the above mentioned items including multivariate extensions to Tsallis statistics and super statistics in non-extensive statistical mechanics.

51. Interpretation of Pathway Parameter in Extended Non-Resonant Thermonuclear Reaction Rate Theory in Astrophysics

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Key words: **Pathway model, Pathway parameter, *H*-function, *G*-function, Non--resonant thermonuclear function, Reaction rate probability integral.**

Abstract

The standard non-resonant thermonuclear reaction rate probability integral in reaction rate theory in Astrophysics is extend by using the pathway model to obtain a wider class of integrals [H. J. Haubold and D. Kumar, *Astroparticle Physics*, **29**(2008), 70-76]. An interpretation of the pathway parameter [A. M. Mathai, *Linear Algebra and Its Applications*, **396**(2005), 317-328] is given in terms of moments. Though the interpretation of the pathway parameter is applicable in any situation but here we consider the extended thermonuclear reaction rate probability integral as an example. Applications of pathway model in other areas are also mentioned.

52. An Empirical Investigation on Detection of Multivariate Outliers in Breeding Data

Prabina Kumar Meher[1], A.R. Rao[2], S.D. Wahi[3] & Namita Jaggi[4]

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Key Words: **Multivariate outliers, Breeding data, Simulation.**

An investigation is carried out to compare the performance of four multivariate outlier detection methods. A multivariate breeding data of maize genotypes obtained from Directorate of Maize Research, New Delhi is used to estimate the mean vectors and dispersion matrices for clean and outlier data. These estimates are used to simulate the samples from clean and outlier populations under different situations, viz. shift outliers or scale outliers or both. All the four methods are compared based on the probability of correctly identified outliers from "outlier data" and probability of wrongly identified outliers from the "clean data" from simulated samples, where each sample is a mixture of known proportion of clean and outlier observations. The method given by Filzmoser *et al.* (2008) is identified as best among all the methods.

53. Estimation of Parameter of Uniform Distribution Based on K-Record Values

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Keywords: **K-record values, Uniform distribution, Maximum likelihood estimation, Best linear unbiased estimation.**

Abstract

There are many situations where experimental outcomes are a sequence of record-breaking observations. In this paper, an extension of record models, well known as k-records, is considered. The best linear unbiased estimator (BLUE) of the parameter of the uniform distribution is derived using k-record values. We have also determined the efficiency of BLUE relative to maximum likelihood estimator (adjusted for bias). The best linear unbiased predictor of future k-record values is also determined.

54. A Stretched Double Exponential Distribution and its Applications in Financial Modeling

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Key Words: **Currency exchange rates, Financial modeling, Gaussian distribution, Laplace law, Mathai's entropy, Statistical mechanics, Symmetric distribution.**

Abstract

The pathway model is introduced by Mathai [2005, Linear Algebra and Its Applications, 396, 317-328] and extensively studied by various authors. The switching property of binomial function to the corresponding exponential form gives the applicability of this distribution in statistical distribution theory and statistical mechanics. This model includes a variety of continuous type distributions like gamma, beta type-l, type-2, Maxwell-Boltzmann density, Gaussian, Laplace, Weibull, Logistic etc. In this paper we introduce a stretched double exponential model we call it as 'q-Laplace distribution' which facilitates a transition to the Laplace distribution as $q \to 1$. We study its properties in detail and show that it can be derived by optimizing the Mathai's entropy subject to certain constraints. Also we consider the distributions of the product and ratio of two random variables coming from the same family of q-Laplace distributions and obtain the resultant distribution in terms of the special functions like G-function and hypergeometric function. We illustrate the use of the q-Laplace distribution in modeling currency exchange rates, which results in a better fit than the classical Laplace distribution.

55. Mittag-Leffler Function, Pathway Model and their Applications in Fractional Calculus

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Key words: **Fractional calculus, Mittag-Leffler function, Pathway model, Reaction-diffusion**.

Abstract

Mittag-Leffler function naturally occurs as a solution of fractional reaction-diffusion problems in physics. A new family of distributions associated with Mittag-Leffler function is introduced. Availability of probability models with thicker or thinner tails through this new density is illustrated. Fractional calculus is a generalization of ordinary differentiation and integration to arbitrary real or complex order. A new fractional operator called pathway fractional

integration operator is established by connecting fractional calculus and pathway model. This operator generalizes the classical Riemann-Liouville fractional integration operator, and also it can be reduced to the Laplace integral transform when the pathway parameter 1. This operator will help to extend classical statistical distributions to the wider classes of distributions which will be more useful in practical applications.

56. Alternative Approach to PPS Sampling using the Concept of Coefficient of Variation when Auxiliary Information is available

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$\alpha \to$

Key words: **Coefficient of mean deviation, Probability of selection, Relative efficiency, Sampling strategy, Sampling design.**

ABSTRACT

A modified sampling strategy is obtained by utilizing the concept of coefficient of variation, when auxiliary information is available. Empirical comparisons suggest that the proposed scheme turns out to be more efficient than the Midzuno Sen scheme.

57. On Estimation of Mean for Positive and Negative Correlation Situations

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Key words: **Coefficient of Variation, Mean Square Error (MSE), Relative Efficiency, Simulation.**

ABSTRACT

For estimating the population mean of a variable of interest class of ratio - product type estimators are investigated by performing efficiency comparisons. Situations are identified under which proposed estimators are better than sample mean and the customary ratio and product estimators. Comparisons are computer-aided empirical studies with Monte-Carlo simulation from hypothetical bivariate normal population. Some artificial populations are also considered which are suitable for positive and negative correlation situations.

58. Power of X- Charts under Measurement Error

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Key words: **Measurement error, Power Function, Testing of Hypothesis X-chart.**

Abstract

The expression for the power of the **X-**chart are derived under measurement error. The power of a control chart under measurement error is examined for the case where both the process average and true variance can change. True and error measurements are additive in nature.

59. Ratio-cum-Product Approach for Estimationof Population Mean Using Information on two Auxiliary Variables

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Key words: **Population mean, Bias, Mean squared error, Correlation Coefficient, Coefficient of kurtosis.**

Abstract

This paper proposes two ratio-cum-product type estimators of finite population mean using known coefficient of kurtosis of auxiliary variates and

correlation coefficient between two auxiliary variates. Proposed estimators have been compared with simple mean estimator, usual ratio estimator, product estimator, estimators proposed by Singh (1965), Singh and Tailor (2005). Almost unbiased estimators have also been obtained using Jackknife technique envisaged by Quenouille (1956) An empirical study is also carried out to demonstrate the performance of the suggested estimators.

2.2 STATISTICS - POSTER PRESENTATIONS

60. Derivation of Optimal Order Quantity under Constraints in Classical Newsboy Setup

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Key Words: **Inventory models, Classical newsboy problem, Optimal order quantity, Constrained optimisation, Non-linear programming.**

Abstract

In this paper we shall consider the classical newsboy problem. The problem is as follows : - A newspaper vendor starts his day with 'q' newspapers in his hand to meet a random demand. At the end of the day he may be left with some excess newspapers in his hands or may face shortage. Accordingly, he has to incur excess or shortage cost. It is a purchase inventory model where quantity received is exactly the same as quantity ordered. We shall here obtain the optimal order quantity by minimizing the expected total cost under exponential demand.

We shall then consider the more interesting situation where optimal order quantity can be derived under some non-trivial constraints. For obtaining the optimal order quantity, apart from simply minimizing the expected total cost, one can introduce some non-trivial constraints while minimizing the expected total cost. For example, we may like to ensure that the demand does not deviate 'too much' from the order quantity with a, high probability. We have derived the optimal order quantity in this setup as well.

Some numerical examples has also been worked out.

61. Reliability and Applications of Type II Compound Laplace Distribution

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Key words: **Reliability, Stress-strength, Type II compound Laplace.**

Abstract

Currently there is lot of interest in the area of stress-strength models, particularly in the estimation of the reliability $R = Pr(X < Y)$, when X and Y are independent random variables belonging to the same univariate family of distributions. This paper describes type II compound Laplace distribution, which is obtained by compounding a Laplace distribution with the gamma distribution. Moment estimators of the three parameters are obtained, various properties are explored and the expression for the reliability R is derived. Finally, applications of type II compound Laplace distribution is highlighted.

62. Statistical Applications in Microarray Technology

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Key words: **cDNA microarray, Differential gene expression, Microarray chip technology.**

Abstract

Microarray is a new powerful tool for studying the expression of thousands of genes simultaneously and it helps in the diagnosis/prognosis of many diseases including cancer. This paper describes the cDNA microarray technology and statistical methods employed in the analysis of differential gene expression data. The principle, methodology and data analysis techniques are presented. The large volume of data generated from microarray experiments need efficient and proper statistical methods for drawing valid conclusions. Some of the major challenges in micro array data, analysis and the potential applications of this emerging technology is discussed.

63. On Characterization of Probability Distributions Using Generalized Entropy Functions

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Key words: **Characterization, Generalized residual entropy, R-norm residual entropy Varma's entropy function, Life time distributions.**

Abstract

Characterization of distributions is an important area in theoretical statistics. It refers to the unique determination of distribution through some assumed properties of certain functionals of the distribution function or properties of statistics based on a random sample drawn from a distribution. In the available literature, a number of results have been devoted to characterizing probability distributions based on various aspects such as moments, conditional distribution, order statistics, record values, reliability measures etc. In the field of reliability theory, we consider mainly life distributions. Here the characterization of distributions is based on various reliability measures such as hazard rate, mean residual life function, residual entropy etc. Shannon's entropy plays an important role in the context of information theory. Since this entropy is not applicable to a system that has survived for some units of time, the concept of residual entropy has been developed in the literature. In the present work, we consider certain generalized entropy measures such as generalized residual entropy of first and second kind, R-norm residual entropy, Varma's entropy function etc. and derive some characterization results for lifetime distributions based on these measures.

64. Double Sampling Plan Under Inspection Error

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Key Words: **Inspection error, Double sampling plan, Poisson process, OC And ASN function.**

ABSTRACT

The effects of inspection errors on acceptance sampling when one is inspecting for the number of nonconformities per item is considered, two types of inspections are defined. Formulas are derived for the OC and ASN for situations when these inspection error are present for double sampling plan. An example illustrates the effects of these inspection errors on both the OC and ASN curves.

65. Row-Column Design Balanced for Nearest Neighbours

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Key Words: **Neighbour effects, Neighbour balanced row-column design, Circular design, Variance balanced.**

Abstract

Experiments in agriculture, horticulture and forestry often show neighbour

effects; that is, the response on a given plot is affected by the treatments on neighbouring plots as well as by the treatment applied to that plot. When treatments are varieties, neighbour effects may be due to differences in height or root vigour. Treatments applied to the crop, such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. In order to avoid the bias in comparing the effects of treatments in this situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbours. Neighbour balanced designs, wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbour(s), are used for these situations. These designs permit the estimation of direct and neighbour effect(s) of treatments. Most of the work on designs incorporating neighbour effects has been done under the block design setting. There may be situations when the heterogeneity present in the experimental material is in two directions and the treatments applied to a unit may be affected by treatments in neighbouring units. Here, row-column model with neighbour effects from adjacent four sides viz. left, right, top and bottom has been considered. The information matrix for estimating the direct and neighbour effects of treatments has been derived. Method of constructing row-column designs balanced for neighbour effects has been developed. The designs obtained are found to be variance balanced for estimating direct effects of treatments.

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R.C.Bose	(1947)
K.B.Madhava	(1946)

Mathematics and Statistics

B.N. Prasad	(1945)
B.M.Sen	(1944)
S.C.Dhar	(1943)

Statistics

V.G.Panse	(1942)

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M.R. Siddiqi	(1941)